

# Generating $\theta_{13}$ from sterile neutrinos in $\mu - \tau$ symmetric models

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## Abstract

The smallness of the  $\theta_{13}$  mixing angle as observed in neutrino oscillation experiments can be understood through an approximated  $\mu - \tau$  exchange symmetry in the neutrino mass matrix. Using recent oscillation neutrino data, but assuming no  $CP$  violation, we study  $\mu - \tau$  breaking parameter space to establish the conditions under which such a breaking could have a perturbative origin. According to the so-obtained conditions, we suggest that a sterile neutrino, matching LSND/MiniBooNE neutrino oscillation results, could provide the necessary ingredients to properly fix atmospheric and  $\theta_{13}$  mixing angles to observable values, without exceeding the sterile neutrino fraction bound in solar oscillations. In such a scenario, we analyze the general effect of a fourth neutrino on the prediction for the effective  $m_{ee}$  majorana mass parameter.

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## I. INTRODUCTION

Neutrino oscillation experiments, using solar, atmospheric, reactor, and accelerator neutrinos, provide compelling evidence in favor of nonzero neutrino masses and mixings [1, 2]. With the exception of LSND [3], MiniBooNe [4], and a recent reanalysis of the flux in some short baseline experiments [5], all existing neutrino oscillation data can be described, and understood, assuming the mixing of only three flavor (standard) neutrinos. Within this framework, data indicate that two of the three neutrino mass eigenstates,  $\nu_{1,2}$ , have a squared mass difference given by  $\Delta m_{21}^2 = m_2^2 - m_1^2 = \Delta m_{sol}^2 \sim 7.5 \times 10^{-5} eV^2$ , whereas the third one,  $\nu_3$ , is separated from the  $\nu_1 - \nu_2$  pair by a splitting given by  $|\Delta m_{31}^2| \sim \Delta m_{ATM}^2 \sim 2.5 \times 10^{-3} eV^2$ . However, the sign in  $\Delta m_{31}^2 = m_3^2 - m_1^2$ , and therefore the neutrino mass hierarchy pattern, is still unknown.

Unlike the quark sector where mixing angles are all small, the measured mixings in oscillation experiments are large, except for  $\theta_{13}$ , which has been found to be rather small. In the standard parametrization, mixings are given by the PontecorvoMakiNakagawaSakata (PMNS) matrix [6, 7],

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \cdot K, \quad (1.1)$$

where  $c_{ij}$  and  $s_{ij}$  stand for  $\cos \theta_{ij}$  and  $\sin \theta_{ij}$ , respectively, of the mixing angles given as  $\theta_{12}$ ,  $\theta_{13}$ , and  $\theta_{23}$ . Here,  $\delta_{CP}$  is the Dirac  $CP$  phase, whereas  $K = \text{Diag}(e^{i\beta_1/2}, e^{i\beta_2/2}, 1)$  is a diagonal matrix containing two Majorana phases which do not contribute to neutrino oscillations. Because of the clear hierarchy in oscillation mass scales, where  $\Delta m_{ATM}^2 \gg \Delta m_{sol}^2$  and the smallness of  $\theta_{13}$ , it is possible to make a direct identification of above mixings with the ones used in a simple two neutrino oscillation analysis of the data. This feature will be useful later on for theoretical approximations. Global fits with all three neutrinos indicate that [1, 2]  $\sin^2 \theta_{12} \equiv \sin^2 \theta_{\odot} \approx 0.308 \pm 0.017$ ,  $\sin^2 \theta_{23} \equiv \sin^2 \theta_{ATM} \approx 0.437_{-0.023}^{+0.033}$  ( $0.455_{-0.031}^{+0.039}$ ), and  $\sin^2 \theta_{13} \approx 0.0234_{-0.0019}^{+0.0020}$  ( $0.0240_{-0.0022}^{+0.0019}$ ), for normal (inverted) hierarchy.  $\delta_{CP}$ , on the other hand, has not been determined well so far.

As in the quark sector, the matrix in Eq. (1.1) actually encodes mixings that are independently used to diagonalize both charged and neutral lepton masses. Nevertheless, it is always possible to rotate any lepton basis into that where both charged lepton masses and

weak interactions are simultaneously diagonal. In such a basis flavor associated to  $e$ ,  $\mu$ , and  $\tau$ , labels became transparent, and, furthermore, the PMNS matrix becomes the one that diagonalizes neutrino masses, given in general by the effective operator

$$(M_\nu)_{\alpha\beta}\bar{\nu}_{\alpha L}(\nu_{\beta L})^c + h.c., \quad (1.2)$$

such that  $U_{PMNS} = U_\nu \cdot K$ . Therefore, the neutrino mass matrix can be written in terms of diagonal (complex) masses,  $M_{diag} = \text{Diag}\{m_1 e^{i\beta_1}, m_2 e^{i\beta_2}, m_3\}$ , simply as

$$M_\nu = U_\nu \cdot M_{diag} \cdot U_\nu^T. \quad (1.3)$$

We will work in such a base hereafter. It is worth noticing that, while the observed  $\theta_{13}$  is close to zero, although non-null,  $\theta_{ATM}$  is close to its maximal value,  $\pi/4$ . Certainly, neither of the central values of these angles is in such critical values; however, it is intriguing to observe that, regardless of the hierarchy, it is possible to establish the approximated empirical relation

$$1/2 - \sin^2 \theta_{ATM} \approx \sin \theta_{13} / \text{few} , \quad (1.4)$$

which suggests that the deviation of  $\theta_{ATM}$  from its maximal value,  $\Delta\theta = \pi/4 - |\theta_{ATM}|$ , could somehow be correlated to the nonzero value of  $\theta_{13}$ . That would be the case if both parameters share the same physical origin. As a matter of fact, in the weak flavor basis we have chosen, it is easy to see that null values of  $\Delta\theta$  and  $\theta_{13}$  do increase the symmetry in the mass neutrino sector, by exhibiting a discrete  $\mu-\tau$  exchange symmetry [9]. As a consequence, observed values of these mixings could be understood as a result of the breaking of  $\mu-\tau$  symmetry. This fact has inspired many theoretical studies in the last years [9–12], but little attention has been paid to exploring models that might provide a physical reason for such a breaking. That is the main question we shall address in the present paper by suggesting the mixing with a fourth sterile neutrino, that also accounts for LSND/MiniBooNE observed oscillations, as the natural source for the violation of  $\mu-\tau$  symmetry. This idea has been explored in Refs. [13, 14], although our general scope in here is quite different.

The paper is arranged as follows. To clearly establish our sterile neutrino hypothesis, we start by revisiting  $\mu-\tau$  symmetry and parametrizing its breaking. Next, we use experimental results on neutrino masses and mixings to explore breaking parameter space, assuming  $CP$  conservation for simplicity, to show that relatively small parameters, and therefore perturbative approximations, are well allowed by the data, provided standard neutrino masses

are almost degenerate. As we will argue, the order of magnitude of such parameters suggests that the naive physical mass scale associated to  $\mu - \tau$  breaking could straightforwardly be identified as the LSND/MiniBooNE scale. Hence, we elaborate a general model for neutrino masses and mixings, including a sterile neutrino, and explore the feasibility that the source of the breaking came from the sterile neutrino sector, the nonsymmetric couplings of which provide for the necessary ingredients to fix all mixings in the model. As we will show, there is indeed a non-null region in parameter space where all experimental observables can be accommodated within one standard deviation. Furthermore, we calculate the sterile fraction in solar neutrinos predicted by the model and discuss the impact of our sterile neutrino model in neutrinoless double beta decay experiments. Finally, we present our conclusions.

## II. NEUTRINO MIXINGS AND $\mu - \tau$ SYMMETRY

First of all, let us remark that in the theoretical limit of null  $\theta_{13}$  and  $\theta_{ATM} = -\pi/4$ , with only three standard flavor neutrinos, there is not a Dirac  $CP$  phase and mixing matrix  $U_\nu$  becomes the bimaximal mixing form

$$U_{BM} = \begin{pmatrix} \cos \varphi_{12} & \sin \varphi_{12} & 0 \\ -\frac{\sin \varphi_{12}}{\sqrt{2}} & \frac{\cos \varphi_{12}}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ -\frac{\sin \varphi_{12}}{\sqrt{2}} & \frac{\cos \varphi_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (2.1)$$

where the only undefined mixing corresponds to  $\varphi_{12}$ , which eventually, upon small corrections, will become the solar mixing. Using this matrix within Eq. (1.3), one can read out the general form of the mass terms, which turn out to be symmetric under the exchange of  $\mu$  and  $\tau$  labels. Indeed, by defining the mass matrix elements as  $m_{\alpha\beta}^0 = (M_\nu)_{\alpha\beta}$ , one obtains

$$\begin{aligned} m_{ee}^0 &= m_1 \cos^2 \varphi_{12} + m_2 \sin^2 \varphi_{12}; \\ m_{e\mu}^0 = m_{e\tau}^0 &= \frac{\sin 2\varphi_{12}}{\sqrt{8}} (m_2 - m_1); \\ m_{\mu\tau}^0 &= \frac{1}{2} (m_1 \sin^2 \varphi_{12} + m_2 \cos^2 \varphi_{12} - m_3); \\ m_{\mu\mu}^0 = m_{\tau\tau}^0 &= \frac{1}{2} (m_1 \sin^2 \varphi_{12} + m_2 \cos^2 \varphi_{12} + m_3); \end{aligned} \quad (2.2)$$

where Majorana phases are to be understood.

Conversely, in the “top-down” approximation, the so-called  $\mu - \tau$  symmetry [9] is expressed as the starting point on mass terms by two general conditions given as  $m_{e\mu}^0 = m_{e\tau}^0$

and  $m_{\mu\mu}^0 = m_{\tau\tau}^0$ , which reduce the number of free mass parameters to 4. Thus, in the limit of exact symmetry, one obtains the predictions for mass eigenvalues

$$\begin{aligned} m_1 &= m_{ee}^0 - \sqrt{2}m_{e\mu}^0 \tan \varphi_{12}, \\ m_2 &= m_{ee}^0 + \sqrt{2}m_{e\mu}^0 \cot \varphi_{12}, \\ m_3 &= m_{\mu\mu}^0 - m_{\mu\tau}^0. \end{aligned} \quad (2.3)$$

where 1 – 2 mixing is given by

$$\tan 2\varphi_{12} = \sqrt{8} \left[ \frac{m_{e\mu}^0}{m_{\mu\mu}^0 + (m_{\mu\tau}^0 - m_{ee}^0)} \right]. \quad (2.4)$$

Besides, null values for  $\theta_{13}$  and  $\Delta\theta$  are predicted. However, as already mentioned, this last is not the case from experimental results. Nevertheless,  $\mu - \tau$  can still be assumed as a rather approximated symmetry in the neutrino sector, such that understanding the sources that contribute to its breaking may enlighten the origin of neutrino mixings. Next, we will elaborate on the parametrization for the breaking of  $\mu - \tau$  symmetry.

In general, any generic neutrino mass matrix can always be parametrized in terms of a symmetric part plus a correction that explicitly breaks the symmetry, by  $M_\nu = M_{\mu-\tau} + \delta M$ , where  $M_{\mu-\tau}$  does posses a  $\mu - \tau$  symmetry, whereas  $\delta M$  is defined by only two nonzero elements,

$$\delta M = \begin{pmatrix} 0 & 0 & \delta \\ 0 & 0 & 0 \\ \delta & 0 & \epsilon \end{pmatrix}, \quad (2.5)$$

where breaking parameters are, clearly, defined as  $\delta = m_{e\tau} - m_{e\mu}$  and  $\epsilon = m_{\tau\tau} - m_{\mu\mu}$ . In this line of thought, understanding the origin of these parameters is a key to understanding  $\theta_{13}$  and  $\theta_{ATM}$ .

Assuming that these are relatively small parameters, in comparison to  $m_{e\mu}$  and  $m_{\mu\mu}$  respectively, observable mixing angles are estimated, in the absence of  $CP$  violation, to satisfy

$$\begin{aligned} \tan 2\theta_\odot &\approx \sqrt{8} \left[ \frac{\bar{m}_{e\mu}}{\bar{m}_{\mu\mu} + (m_{\mu\tau} - m_{ee})} \right]; \\ \sin \theta_{13} &\approx \frac{1}{\sqrt{8}} \left[ \frac{2m_{\mu\tau}\delta - \epsilon\bar{m}_{e\mu}}{\bar{m}_{e\mu}^2 + m_{\mu\tau}(\bar{m}_{\mu\mu} - m_{\mu\tau} - m_{ee})} \right]; \\ \sin \Delta\theta &\approx \frac{1}{4} \left[ \frac{\epsilon(m_{\mu\tau} + m_{ee} - \bar{m}_{\mu\mu}) + 2\bar{m}_{e\mu}\delta}{\bar{m}_{e\mu}^2 + m_{\mu\tau}(\bar{m}_{\mu\mu} - m_{\mu\tau} - m_{ee})} \right], \end{aligned} \quad (2.6)$$

where  $\bar{m}_{e\mu} \equiv (m_{e\mu} + m_{e\tau})/2$  and  $\bar{m}_{\mu\mu} \equiv (m_{\mu\mu} + m_{\tau\tau})/2$ . Notice that for small  $\Delta\theta$ , one would have that  $\sin \Delta\theta \approx 1/2 - \sin^2 \theta_{ATM}$ , which jointly to  $\sin \theta_{13}$  would be given by linear relations in terms of  $\epsilon$  and  $\delta$ . Of course, the former expressions are first-order calculations that would provide a good approximation, provided the breaking parameters are small enough. It is remarkable, though, that solar mixing turns out to have a similar expression to that obtained in the exact symmetric limit.

Since we already have quite more precise information about the mixing angles, it seems interesting to look at the parameters the other way around, by addressing the theoretical question regarding how good  $\mu - \tau$  is as an approximated symmetry, that is, to obtain information about the relative size of the breaking parameters, as a way to search for hints of any possible physics lying beneath them. In particular, for instance, knowing to what extent  $\delta M$  could be treated as a perturbation could give a hint toward knowing how far in the energy scale the breaking source lies away from the overall active neutrino mass scale. This possibility is in itself an interesting one, and our main goal on the following discussion will be to explore under which conditions one could achieve a perturbative breaking of  $\mu - \tau$ , meaning acceptable small values for the breaking parameters  $\epsilon$  and  $\delta$ .

Early work has shown that the source of such a breaking cannot come from within the Standard Model physics, where the only breaking source is the  $\mu$  and  $\tau$  mass difference [11]. As a matter of fact, this charged lepton mass difference is indeed communicated through charged weak interactions to the neutrino sector, becoming, upon radiative corrections, a source for nonzero  $\delta M$ . Nevertheless, such a correction turns out to be too small to account for observed mixings. Therefore, we are moved to assume that there should be a breaking sector out of the Standard Model.

Without relying on any approximation, one could make a direct reconstruction of the mass matrix in Eq. (1.3) and thus of the actual values for  $\delta$  and  $\epsilon$  parameters. To this aim, however, one would require knowledge of the mass spectrum, which we do not have so far. What we do have, instead, are the two values for mass squared differences involved in neutrino oscillations,  $\Delta m_{sol}^2$  and  $\Delta m_{ATM}^2$ . Thus, one mass parameter in the spectrum, which we take as the lightest absolute neutrino mass, aside from the relative sign of mass eigenvalues, would remain as free parameters. Notice that by the last we mean to take Majorana phases to be either 0 or  $\pi$  so that they provide just a relative sign for the masses. Dirac  $CP$  phase we will assume hereafter to be zero. In these terms, we rewrite the absolute

mass eigenvalues as

$$\begin{aligned} |m_2| &= \sqrt{m_0^2 + \Delta m_{sol}^2} \quad \text{and} \quad |m_3| = \sqrt{m_0^2 + |\Delta m_{ATM}^2|} \quad \text{for NH.} \\ |m_1| &= \sqrt{m_0^2 + |\Delta m_{ATM}^2|} \quad \text{and} \quad |m_2| = \sqrt{m_0^2 + |\Delta m_{ATM}^2| + \Delta m_{sol}^2} \quad \text{for IH.} \end{aligned} \quad (2.7)$$

Note that, in above, the lightest mass eigenstate,  $m_0$ , becomes  $m_1$  for the normal mass hierarchy (NH) and  $m_3$  for the inverted mass hierarchy (IH). Next, to proceed with our analysis, we define, without approximations, the dimensionless parameters

$$\begin{aligned} \hat{\delta} &\equiv \frac{\delta}{m_{e\mu}} = \frac{\sum_i (U_{ei}U_{\tau i} - U_{ei}U_{\mu i})m_i}{\sum_i U_{ei}U_{\mu i}m_i} \\ \hat{\epsilon} &\equiv \frac{\epsilon}{m_{\mu\mu}} = \frac{\sum_i (U_{\tau i}U_{\tau i} - U_{\mu i}U_{\mu i})m_i}{\sum_i U_{\mu i}U_{\mu i}m_i}, \end{aligned} \quad (2.8)$$

where the right-hand-sides have been written according to Eq. (1.3). Combined with Eq. (2.7), the last expressions give the dimensionless parameters in terms of observed mixing angles, oscillation mass scales, and the absolute scale of neutrino masses,  $m_0$ , as the only free parameter. Next, let us perform an approximated analytical analysis of the expressions in Eq. (2.8) by considering all four independent combinations of mass signs: (i)  $m_{1,2,3} > 0$ , (ii)  $m_{1,2} < 0$  but  $m_3 > 0$ , (iii)  $m_{1,3} > 0$  but  $m_2 < 0$ , and (iv)  $m_1 < 0$  but  $m_{2,3} > 0$ . Those can be written in the suitable form

$$\begin{aligned} \hat{\delta} &= \frac{y_- f s_{13} + y_+}{1 + f s_{13} \tan \theta_{23}}, \\ \hat{\epsilon} &= \frac{g \cos 2\theta_{23} - s_{13}h}{1 + g s_{23}^2 + s_{13}h/2}, \end{aligned} \quad (2.9)$$

where

$$\begin{aligned} y_{\pm} &= \frac{c_{23} \pm s_{23}}{c_{23}}, \\ f &= \frac{m_1 c_{12}^2 + \sigma m_2 s_{12}^2 - \Sigma m_3}{c_{12} s_{12} (m_1 - \sigma m_2)}, \\ g &= \frac{m_1 (c_{12}^2 s_{13}^2 - s_{12}^2) + \sigma m_2 (s_{12}^2 s_{13}^2 - c_{12}^2) + \Sigma m_3 c_{13}^2}{m_1 s_{12}^2 + \sigma m_2 c_{12}^2}, \\ h &= \frac{(m_1 + \sigma m_2) \sin 2\theta_{23} \sin 2\theta_{12}}{m_1 s_{12}^2 + \sigma m_2 c_{12}^2}, \end{aligned} \quad (2.10)$$

with the conventions  $\sigma = +$ ,  $\Sigma = \pm$  for cases i and ii and  $\sigma = -$ ,  $\Sigma = \pm$  for cases iii and iv, respectively. As one can see from these expressions,  $\hat{\delta}$  and  $\hat{\epsilon}$  in Eq. (2.9) become zero when  $\theta_{13} = 0$  and  $\theta_{23} = -\pi/4$ , as expected from exact  $\mu - \tau$  symmetry. In the following, let us

first examine under which considerations  $\hat{\delta} \ll 1$ , and latter on, we will analyze the behavior of  $\hat{\epsilon}$  in such cases. In the three approaches given by the hierarchies, and using the central values for the current mixing parameters, we have

- For NH,  $m_1 \ll m_2 \approx \sqrt{\Delta m_{sol}^2} \ll m_3 \approx \sqrt{\Delta m_{ATM}^2}$ , and thus

$$f \approx \frac{\Sigma}{\sigma s_{12} c_{12}} \sqrt{\frac{\Delta m_{ATM}^2}{\Delta m_{sol}^2}} \left( 1 - \frac{\sigma s_{12}^2}{\Sigma} \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{ATM}^2}} \right), \quad |f| \sim 12.5, \quad (2.11)$$

which implies  $|\hat{\delta}| \sim 3.26$ , discarding NH for any mass sign combinations.

- For IH,  $m_1 \approx \sqrt{\Delta m_{ATM}^2}$ ,  $m_2 \approx \sqrt{\Delta m_{sol}^2 + \Delta m_{ATM}^2} \gg m_3$ , which gives

$$f \approx \frac{c_{12}^2 + \sigma s_{12}^2 + \frac{\sigma s_{12}^2}{2} \frac{\Delta m_{sol}^2}{\Delta m_{ATM}^2}}{s_{12} c_{12} \left( 1 - \sigma - \frac{\sigma}{2} \frac{\Delta m_{sol}^2}{\Delta m_{ATM}^2} \right)}. \quad (2.12)$$

For cases i and ii, we have  $|f| \sim 10^2$ , such that  $|\hat{\delta}| \sim 2$ , whereas in cases iii and iv, we obtain  $|f| \sim 1$  and hence  $|\hat{\delta}| \sim 0.1$ . Therefore, cases i and ii again seem to be ruled out.

- Finally, for degenerated hierarchy (DH), we get

$$f \approx \frac{c_{12}^2 + \sigma s_{12}^2 - \Sigma - \frac{\Sigma}{2} \frac{\Delta m_{ATM}^2}{m_0^2}}{c_{12}^2 s_{12}^2 \left( 1 - \sigma - \frac{\sigma}{2} \frac{\Delta m_{sol}^2}{m_0^2} \right)}. \quad (2.13)$$

Cases i and ii give  $|f| \gtrsim 10^2$ , which now implies  $|\hat{\delta}| \gtrsim 15$ , while for cases iii and iv, one gets  $|f| \lesssim 1$  and  $|\hat{\delta}| \lesssim 0.1$ . Therefore, in DH, cases i and ii are once more disfavored.

The approximations taken above suggest that only cases iii and iv, in the IH and DH, allow for small values of  $\hat{\delta}$ . In the IH, a similar analysis, after some algebra, gives  $\hat{\epsilon} \sim 1$ , for the iii and iv combinations, whereas in the DH, we obtained  $\hat{\epsilon} \sim 1$  for case iii and  $\hat{\epsilon} \sim 0.4$  for case iv. Thus, our analysis indicates that the only fairly perturbative case would occur in the DH for the signs combination that corresponds to  $m_1 < 0$  and  $m_{2,3} > 0$ .

After a complete numerical analysis of the parameter space allowed by data (without any approximation), in all four mass sign independent combinations, it was found that, while in all possible cases there is always a solution with nonzero values for either of, or simultaneously both, the  $\hat{\delta}$  and  $\hat{\epsilon}$  parameters, the only case one might consider as fairly



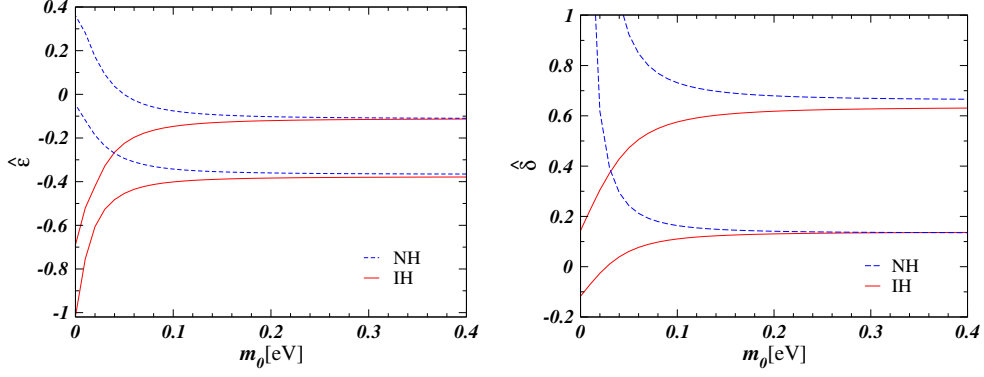


FIG. 1: One sigma regions for the allowed values of dimensionless  $\mu - \tau$  breaking parameters,  $\hat{\epsilon}$  and  $\hat{\delta}$ , as a function of the lightest neutrino mass,  $m_0$ , for inverted (red line) and normal (blue dashed line) hierarchy, in the case where  $m_1 < 0$ ,  $m_{2,3} > 0$ .

perturbative corresponds to  $m_1 < 0$  and  $m_{2,3} > 0$  (regardless of the hierarchy). This is consistent with our previous analysis. The allowed one sigma region for both breaking parameters in this case is depicted in Fig. 1. Moreover, as one can see from this figure, only for almost degenerate neutrinos, where  $m_0 > 0.1$  eV, is it possible to actually pick up relatively small values for  $\hat{\delta}$  and  $\hat{\epsilon}$  to comply with the expectation of a perturbative origin. Interestingly enough, none of the breaking parameters is null within such an allowed region. We must mention that our results are consistent with those obtained in the general analysis made in Ref. [12], although our general scope here is quite different.

Naively, if one takes, for instance,  $\hat{\epsilon} \simeq \hat{\delta} \simeq 0.2$  or so, valid in the whole region for  $m_0$  above 0.1 eV for  $m_0$ , the approximation we used to derive the mixings in Eq. (2.6) would be quite well justified, and so would be, to the numerical extent, the geometrical relation among mixings given in Eq. (1.4).

### III. $\mu - \tau$ BREAKING FROM A STERILE NEUTRINO

There are at least two possible approximations one can make to explore the physics beyond standard model that is responsible of generating the breaking of  $\mu - \tau$  symmetry. Either this lies close to the same physics that is responsible for the smallness of neutrino masses, in which case one has to probably go for model building to explore concrete possibilities, or it is the consequence of the mixing with a sector that does not comply with the symmetry.

An example of the latter is the mixing corrections induced through radiative processes and due to the explicit violation of the symmetry in the charged lepton masses. As mentioned already, this is too small to account for the observed effect in neutrino mixings. Another quite straightforward candidate for this would be a sterile neutrino, which by definition does not have weak interactions, and thus it has no (e, mu, or tau) lepton flavor. This last possibility is much more intriguing, because the not-so-small parameters that are required to understand the mixings do suggest that such a new sector cannot be too far away from the standard neutrino mass scale. Actually, by assuming that the breaking parameters are somehow generated at a given larger scale,  $m_s$ , and naively taking the perturbations that break the symmetry as given in terms of the ratio among the involved scales, which means that  $\hat{\epsilon}, \hat{\delta} \approx m_\nu/m_s$ , then the mass scale of the sterile neutrino should be just about the  $eV$  scale, precisely as suggested by LSND/MiniBooNe results. Next, we will analyze in detail such a possibility.

To be specific in our analysis, we assume a single light sterile neutrino and consider its most general mass terms, including the mixing with the standard active neutrino sector. Notice, however, that we shall be working in a  $3 + 1$  neutrino mixing scheme in which the fourth neutrino (predominantly sterile) is isolated from the block of three active flavor neutrinos by the mass gap  $\Delta m_{LSND}^2 \approx (0.4 - 10) \text{ eV}^2$ . Hence, in the basis  $(\nu_e, \nu_\mu, \nu_\tau, \nu_s)$ , the mass matrix can be written as

$$\mathcal{M} = \begin{pmatrix} M_{\mu-\tau} & \vec{\alpha} m_s \\ \vec{\alpha}^T m_s & m_s \end{pmatrix}, \quad (3.1)$$

where  $m_s$  is the Majorana mass for the sterile neutrino and the vector  $\vec{\alpha}^T = (\alpha_e, \alpha_\mu, \alpha_\tau)$  denotes the active-sterile mixing masses in units of  $m_s$ . Specific structures of this vector could have consequences for model builders, as discussed in Ref. [14]. Next, let us assume that  $\alpha_\ell \ll 1$  and  $m_s \gg m_{\alpha\beta}$ , where  $m_{\alpha\beta}$  are the elements of the active and explicitly symmetric flavor matrix  $M_{\mu-\tau}$ . Clearly, if  $\alpha_\mu = \alpha_\tau$ , the whole sector would be invariant under  $\mu - \tau$  symmetry, with the known consequences of it for active neutrino mixings. We will not assume so, and thus the model will have a single effective parameter for the breaking of the symmetry given by the coupling differences  $\Delta\alpha = \alpha_\tau - \alpha_\mu$ . Nevertheless, one would find it useful to keep track of the independent  $\alpha$ 's along the calculations.

After decoupling  $\nu_s$ , we get, at the lower-order approximation, an effective active flavor

matrix,  $M_\nu$ , the elements of which are given by the (low-energy) seesaw formula,

$$(M_\nu)_{\rho\delta} \simeq (M_{\mu-\tau})_{\rho\delta} - \alpha_\rho m_s \alpha_\delta^T. \quad (3.2)$$

It is clear that  $M_\nu$  does not possess in general  $\mu-\tau$  symmetry due to the presence of the term  $\alpha_\rho m_s \alpha_\delta^T$ . It is important to notice that the last can always be separated in a symmetric plus a nonsymmetric part under the exchange of  $\mu$  and  $\tau$  indexes. The symmetric part, however, will only account for corrections, of second order in  $\alpha_\ell$ , to the mass spectrum and the solar mixing angle defined by  $M_{\mu-\tau}$  alone. On the other hand, the nonsymmetric part would be the source for the breaking parameters defined in the previous section. It is actually easy to see that, without further approximations, one gets  $\delta = \alpha_e \Delta \alpha m_s$  and  $\epsilon = 2\bar{\alpha}_\mu \Delta \alpha m_s$ , where, as before,  $\bar{\alpha}_\mu = (\alpha_\mu + \alpha_\tau)/2$ . It is straightforward to show that, regardless of hierarchy, our now effective dimensionless breaking parameters are second order in  $\alpha_\ell$ , and for quasidegenerate neutrinos, they can be approximated as

$$\begin{aligned} \hat{\delta} &\approx \frac{\sqrt{2} m_s \alpha_e \Delta \alpha}{m_0 \sin 2\varphi_{12}}, \\ \hat{\epsilon} &\approx \frac{2 m_s \bar{\alpha}_\mu \Delta \alpha}{m_0 \cos^2 \varphi_{12}}, \end{aligned} \quad (3.3)$$

where the mixing  $\varphi_{12}$  is the one involved in the diagonalization of the symmetric sector. Within a rough approximation, at lower order, one would have  $\varphi_{12} \sim \theta_\odot$ . Therefore, to get an idea of the order of magnitude of the sterile to active neutrino couplings, one may take, for instance,  $m_s \sim 1 \text{ eV}$ ,  $m_o \sim 0.2 \text{ eV}$ , and  $\hat{\epsilon} \sim \hat{\delta} \sim 0.2$  which are consistent with the analysis in previous sections, to show that a solution the above formulas is found for  $\alpha_e \sim 0.23$ ,  $\bar{\alpha}_\mu \sim 0.16$ , and  $\Delta \alpha \sim 0.11$ . Notice, however, that the effect of  $\alpha_e$ , even for  $\Delta \alpha = 0$ , is to incorporate corrections to the mixing in the  $1-2$  sector, and thus, a more accurate calculation is likely to modify these naive estimates.

Notice that, in getting the above results, it seems that three  $\alpha_\ell$  couplings do contribute to only two effective breaking parameters,  $\hat{\delta}$  and  $\hat{\epsilon}$ . Nevertheless, we would expect that any physical solution should at least be around above roughly estimated values for  $\alpha_{\mu,\tau}$ . To address this issue in a more reliable way, one should explicitly confront mass scales and mixing angles as obtained by the diagonalization of the complete  $3+1$  neutrino sector against measured experimental parameters. To this aim, let us first point out that the mass matrix given in Eq. (3.1) contains eight independent parameters ( $m_{\ell\ell'}$ ,  $\alpha_\ell$ , and  $m_s$ ), whereas we have knowledge of seven experimentally determined observables, enumerated as follows.

From weak flavor oscillations one gets two squared mass scales,  $\Delta m_{ATM}^2$  and  $\Delta m_{sol}^2$ , and three mixing angles,  $\theta_\odot$ ,  $\theta_{ATM}$ , and  $\theta_{13}$ . Additionally, from LSND/MiniBooNe results, one gets two parameters, taken as a squared mass scale  $\Delta m_{LSND}^2 \approx \Delta m_{s\ell}^2 \approx m_s$  and a mixing,  $\theta_{e\mu}$ . Therefore, there would be only one free parameter in the analysis, which we take as the lightest neutrino mass scale,  $m_0$ . As already discussed, the consistency of our model with a perturbative treatment of the breaking of  $\mu - \tau$  symmetry requires  $m_0$  to be within 0.1 to 0.4 eV, which corresponds to degenerated hierarchy. This short range for  $m_0$  will end up narrowing the allowed parameter space, as we will show below.

Next, for our analysis, we will take the intermediate neutrino mass eigenvalues as given in terms of the atmospheric and solar scales by Eq. (2.7). Moreover, following the outcome of the previous discussion, and considering a perturbative diagonalization of  $\mathcal{M}$ , one can see that active mass eigenvalues are well approximated (at lower order) by the eigenvalues of  $M_{\mu-\tau}$ , given in Eq. (2.3), whereas  $m_4 \approx m_s$ . This leave us only with the question of constructing a self-consistent system of equations to fit all experimental mixing angles with the remaining parameters of the model. By considering the relevant effective oscillations in solar, atmospheric, and short baseline experiments, one gets the general formulas

$$\sin^2 2\theta_\odot = 4|\mathcal{U}_{e1}|^2|\mathcal{U}_{e2}|^2, \quad (3.4)$$

$$\sin^2 2\theta_{13} = 4|\mathcal{U}_{e3}|^2(|\mathcal{U}_{e1}|^2 + |\mathcal{U}_{e2}|^2), \quad (3.5)$$

$$\sin^2 2\theta_{ATM} = 4|\mathcal{U}_{\mu3}|^2|\mathcal{U}_{\tau3}|^2, \quad (3.6)$$

$$\sin^2 2\theta_{e\mu} \simeq 4|\mathcal{U}_{e4}|^2|\mathcal{U}_{\mu4}|^2, \quad (3.7)$$

where  $\mathcal{U}_{\alpha i}$ , for  $i = 1, 2, 3, 4$ , and  $\alpha = e, \mu, \tau, s$ , stands for the elements of the general mixing matrix which diagonalizes  $\mathcal{M}$ . As it is well known, since we are neglecting  $CP$  violation, the columns of  $\mathcal{U}$  are given by the properly normalized eigenvectors and  $\mathcal{M}$ . Here, we emphasize that the left-hand (lhs) sides of Eqs. (3.4-3.6) are known from usual neutrino oscillation experimental data, whereas the lhs of Eq. (3.7) comes from considering the allowed regions of LSND and Mini-BooNE neutrino data [8], which we take as

$$\sin^2 2\theta_{e\mu} = 0.0023, \quad |\Delta m_{41}^2| = 0.89 eV^2. \quad (3.8)$$

On the other hand, the entries in rhs of Eqs. (3.4-3.7), are given up to  $\mathcal{O}(\alpha^2)$  by the following expressions:

$$\mathcal{U}_{e1} \approx c_{12} + s_{12} \frac{m_4^2}{m_{12} m_{14}} \alpha_+ \alpha_- - \frac{c_{12}}{2} \frac{m_4^2}{(m_{14})^2} \alpha_-^2, \quad (3.9)$$

$$\mathcal{U}_{e2} \approx s_{12} + c_{12} \frac{m_4^2}{m_{21} m_{24}} \alpha_+ \alpha_- - \frac{s_{12}}{2} \frac{m_4^2}{(m_{24})^2} \alpha_+^2, \quad (3.10)$$

$$\mathcal{U}_{e3} \approx \frac{\Delta\alpha}{\sqrt{2}} \frac{m_4^2}{m_{34}} \left[ \frac{c_{12}}{m_{31}} \alpha_- + \frac{s_{12}}{m_{32}} \alpha_+ \right], \quad (3.11)$$

$$\mathcal{U}_{e4} \approx \frac{m_4}{m_{41}} \alpha_e, \quad (3.12)$$

$$\mathcal{U}_{\mu3} \approx -\frac{1}{\sqrt{2}} + \frac{\Delta\alpha}{2} \frac{m_4^2}{m_{34}} \left[ \frac{c_{12}}{m_{32}} \alpha_+ - \frac{s_{12}}{m_{31}} \alpha_- \right] + \frac{(\Delta\alpha)^2}{4\sqrt{2}} \frac{m_4^2}{(m_{34})^2}, \quad (3.13)$$

$$\mathcal{U}_{\mu4} \approx \frac{m_4}{m_{41}} \alpha_\mu, \quad (3.14)$$

$$\mathcal{U}_{\tau3} \approx \frac{1}{\sqrt{2}} + \frac{\Delta\alpha}{2} \frac{m_4^2}{m_{34}} \left[ \frac{c_{12}}{m_{32}} \alpha_+ - \frac{s_{12}}{m_{31}} \alpha_- \right] - \frac{(\Delta\alpha)^2}{4\sqrt{2}} \frac{m_4^2}{(m_{34})^2}. \quad (3.15)$$

Here, to simplify, we have introduced the shorthand notation  $m_{ij} = m_i - m_j$ , for  $i, j = 1 \dots 4$ ,  $\alpha_+ = \alpha_e s_{12} + \sqrt{2} c_{12} \bar{\alpha}_\mu$  and  $\alpha_- = \alpha_e c_{12} - \sqrt{2} s_{12} \bar{\alpha}_\mu$ , where, as before,  $c_{12}$  ( $s_{12}$ ) stands for the cosine (sine) function of the free parametric angle,  $\varphi_{12}$ , defined by Eq. (2.4).

As it is easy to see, one can use LSND/MiniBooNe mixing in order to solve for  $\alpha_e$  in terms of  $\alpha_\mu$ , using Eq. (3.7). In the quasidegenerate neutrino scenario with sterile mass dominance that we are considering, this implies that  $\sin 2\theta_{e\mu} \approx 2|\alpha_\mu \alpha_e|$ . Numerically, this means that  $|\alpha_\mu \alpha_e| \approx 0.02$ . Similarly, in the same approximation, we obtain for the solar mixing

$$\sin^2 2\theta_\odot \approx \sin^2 2\varphi_{12} \cdot \left[ 1 - (\alpha_e^2 + 2\bar{\alpha}_\mu^2) + \frac{m_s}{m_0} \left( \cos 2\varphi_{12} (\alpha_e^2 - 2\bar{\alpha}_\mu^2) + \sqrt{8} \alpha_e \bar{\alpha}_\mu \cot 2\varphi_{12} \right) \right],$$

regardless of the hierarchy. It is worth noticing that the last expression does depend on four effective parameters,  $m_0$ ,  $m_4$ , and  $\alpha_{\mu,\tau}$ . In practice, since we are choosing  $m_0$  in a given interval, this relation can be used to formally fix  $\varphi_{12}$  mixing from the equations system, leaving us with only two relevant independent parameters:  $\alpha_\mu$  and  $\alpha_\tau$  couplings. Finally, these last parameters can be estimated (at least formally) from the formulas that give  $\theta_{13}$  and atmospheric mixings, in Eqs. (3.5) and (3.6), which at the lower order in the  $\alpha$  parameters are written as

$$\sin^2 2\theta_{13} \approx \left( \frac{4\Delta\alpha m_s m_0}{\Delta m_{ATM}^2} \right)^2 \left[ \alpha_e \left( \frac{\Delta m_{ATM}^2}{4m_0^2} \pm \sin^2 \varphi_{12}^2 \right) \pm \frac{\bar{\alpha}_\mu \sin 2\varphi_{12}}{\sqrt{2}} \right]^2, \quad (3.16)$$

$$\sin^2 2\theta_{ATM} \approx 1 - (\Delta\alpha)^2. \quad (3.17)$$

The sign difference in Eq. (3.16) stands for normal and inverted hierarchies, respectively. Notice that  $\sin^2 2\theta_{13}$  comes from corrections at the fourth order in  $\alpha$ 's, although second order in  $\Delta\alpha$ , whereas atmospheric mixing gets a second-order correction, as suggested by the naive numerical expression in Eq. (1.4).

Once we have some understanding of the parameter correlations in the determination of the four observable mixings given in Eqs. (3.4) to (3.7), we can now proceed with a numerical analysis of such a set of equations without further approximations, in order to explore and identify the allowed parameter space for  $\alpha_\mu$  and  $\alpha_\tau$  that gives consistent results for current experimental oscillation neutrino data, within one sigma deviations. Our results, for  $m_0 = 0.2 \text{ eV}$  and both the hierarchies, are presented in Fig. 2, where we have scanned for appropriated values of  $\alpha_\mu$  and  $\alpha_\tau$  parameters for each mixing as independent, such that the consistent values are found in the overlapping of all regions (shaded area in the given plots) that give one sigma values for each standard mixing angle. In these same plots, we have also constrained the regions such that  $-0.4 \lesssim \hat{\epsilon} \lesssim 0.3$  and  $0.1 \lesssim \hat{\delta} \lesssim 0.6$ , to insure that the whole allowed parameter space be consistent with perturbative approximations. The actual effect of including this last condition is to bound the allowed parameter space from the bottom and the top, as it can be seen on the given plots.

By picking up some allowed values for the  $\alpha$  parameters, it is easy to reconstruct the four-by-four mass matrix to give an explicit numerical example for it. For a typical mass matrix obtained by this procedure, we consider

$$\mathcal{M} = \begin{pmatrix} 0.0247 & 0.0110 & 0.0110 & -0.1881 \\ 0.0110 & 0.0489 & -0.0205 & 0.1315 \\ 0.0110 & -0.0205 & 0.0489 & 0.0343 \\ -0.1881 & 0.1315 & 0.0343 & 0.9428 \end{pmatrix}, \quad (3.18)$$

for  $\alpha_e = -0.1995$ ,  $\alpha_\mu = 0.1395$ , and  $\alpha_\tau = 0.0364$ . This matrix leads to the exact active neutrino mixings,  $\sin^2 \theta_\odot = 0.280$ ,  $\sin^2 \theta_{ATM} = 0.379$ , and  $\sin^2 \theta_{13} = 0.021$ , which are in good agreement with neutrino oscillation measured parameters, within two, three, and two sigma deviations, respectively. Furthermore, we get for the LSND/MiniBooNe mixing  $\sin^2 \theta_{e\mu} = 0.002$ , in agreement with the fits of short-baseline neutrino oscillation data [8]. On the other hand, the corresponding squared mass differences obtained out of this example are  $\Delta m_{sol}^2 = 7 \times 10^{-5} \text{ eV}^2$  and  $\Delta m_{ATM}^2 = 2.6 \times 10^{-3} \text{ eV}^2$ , whereas we get for the sterile mass eigenvalue  $m_s = 0.997 \text{ eV}$ , which are also consistent with observations. More accurate

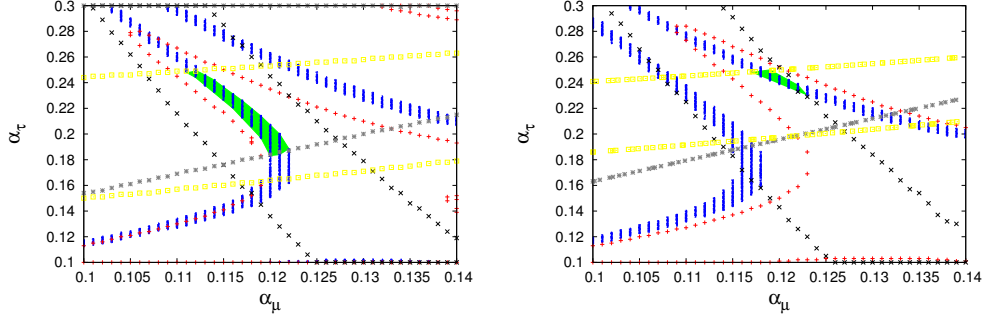


FIG. 2: (Color on line) Allowed parameter space for  $\alpha_\mu$  and  $\alpha_\tau$  that is consistent with all oscillation neutrino data (shaded region), in the 3+1 scenario, within one sigma deviation for normal (lhs) and inverted (rhs) hierarchy, respectively. The dotted (blue) region corresponds to the bounds given by  $\sin^2 2\theta_{13}$ , whereas (red) crossed curves delimit the region for acceptable results of  $\sin^2 2\theta_{ATM}$ . The one sigma  $\sin^2 \theta_\odot$  region is bounded by (black) ex-marked curves. (Yellow) squared curves and (gray) star lines bound the regions where  $\hat{\epsilon}$  and  $\hat{\delta}$  are small enough for the model to be perturbative.

results could be obtained if higher-order corrections in  $\alpha$  parameters are incorporated in the reconstruction of the mass matrix. Nevertheless, this numerical matrix serves as a good example to illustrate the mechanism we are exploring.

#### IV. STERILE IMPACT ON OTHER NEUTRINO OBSERVABLES

SK and SNO experiments have measured solar electron neutrino flux,  $\Phi_{\nu_e}$ , whereas SNO has also measured the total solar flux of active neutrinos,  $\Phi_{\nu_e, \nu_\mu, \nu_\tau}$ , using neutral current interactions. These measurements are in good agreement with the total neutrino flux,  $\Phi_B$ , predicted by the solar model (see Ref. [2] for further references), which can be used to constrain solar neutrino conversion into sterile neutrinos. Thus, assuming that solar neutrinos oscillate as  $\nu_e \rightarrow \sin \alpha \nu_s + \cos \alpha \nu_{\mu, \tau}$ , the sterile fraction  $\eta_s \equiv \sin^2 \alpha$  is estimated to be [16]

$$\eta_s \approx \frac{\Phi_B - \Phi_{\nu_e, \nu_\mu, \nu_\tau}}{\Phi_B - \Phi_{\nu_e}} \approx 0 \pm 0.2 . \quad (4.1)$$

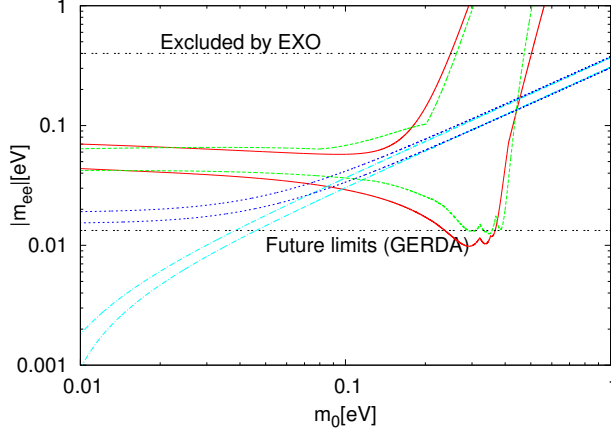


FIG. 3: (Color online) Allowed ranges of the effective mass  $|m_{ee}|$  as a function of the lightest neutrino mass  $m_0$  in the case of normal (red and light blue) and inverted (green and blue) neutrino mass hierarchy. The lower narrowed regions (delimited by the light blue and blue lines) correspond to the conventional case of three active neutrinos. Wider upper regions (delimited by red and green lines) correspond to our model.

Theoretically,  $\eta_s$  can be estimated in our model by using  $\eta_s = \frac{P_{\nu_e \rightarrow \nu_s}}{1 - P_{\nu_e \rightarrow \nu_e}}$ . In terms of mixing matrix elements, and considering only the contributions at the solar scale, one can write

$$\eta_s \approx -\frac{4U_{e1}U_{e2}U_{s1}U_{s2}}{4[U_{e1}U_{e2}]^2}. \quad (4.2)$$

Taking values within the allowed parameter regions for  $\alpha_\mu$  and  $\alpha_\tau$ , presented in the previous section, for  $m_0 = 0.2 \text{ eV}$ , we found that  $\eta_s \approx (1.2 - 1.9) \times 10^{-2}$  for normal hierarchy, whereas  $\eta_s \approx (2.7 - 3) \times 10^{-2}$  for inverted hierarchy. Clearly, this results agree with the bounds given in Eq. (4.1).

On the other hand, our sterile neutrino model will also have direct and interesting implications on the effective Majorana mass term that is involved in neutrinoless double beta decay, now written as

$$|m_{ee}| = \left| \sum_{i=1}^4 U_{ei}^2 m_i \right|. \quad (4.3)$$

The allowed values of  $|m_{ee}|$  in our model can be calculated as a function of the lightest neutrino mass. Our results are shown in Fig. 3, where, as before, we have used one sigma values for oscillations parameters. Consistently, sterile parameters were considered in the regions given as  $0.1 \leq \alpha_\mu \leq 0.14$ ,  $0.2 \leq \alpha_\tau \leq 0.23$  for normal hierarchy and  $0.12 \leq \alpha_\mu \leq$



0.14,  $0.24 \leq \alpha_\tau \leq 0.26$  for inverted hierarchy. This ranges are consistent with neutrino oscillation data in the region where  $0.2 \leq m_0 \leq 0.4$ , which corresponds to the degenerated neutrino mass hierarchy.

As it can be seen, the allowed parameter in our model region is enhanced, compared to three-neutrino case, due to the presence of the sterile neutrino. However, notice that mass hierarchy makes little difference for the allowed parameter space, and thus, it would be difficult to be experimentally identified. Nevertheless, other interesting differences are at hand. In particular, if forthcoming experiments were to observe a positive signal between 0.01-0.4 eV, a degenerated mass spectrum with  $|m_0|$  between 0.2 and 0.4 eV might still be possible in the case of four neutrinos. On the other hand, the nonobservation of a signal in experiments like GERDA [17] would practically rule out this model, which makes it falsifiable.

## V. CONCLUDING REMARKS

Neutrino mass models based on  $\mu - \tau$  symmetry remain as an interesting possibility since they can provide a natural understanding for the almost maximal value of atmospheric neutrino mixing, and the smallness of reactor  $\theta_{13}$  mixing, using only a couple of generic parameters that encode the breaking of the symmetry. As the analysis shows, current neutrino data are consistent with small values for such parameters, although, in such a scenario, it seems to prefer a quasidegenerate active neutrino spectrum. As we have also pointed out, the perturbative regime of  $\mu - \tau$  symmetry breaking also provides an understanding of the, otherwise accidental, relation among atmospheric and  $\theta_{13}$  mixings, which can be expressed through the phenomenological numerical formula  $1/2 - \sin^2 \theta_{ATM} \approx \sin \theta_{13}/\text{few}$ . As it turns out, from our discussion, at the lower-order approximation, both sides of this equation are given as linear expressions in terms of the symmetry breaking parameters.

On the other hand, the relative smallness of the breaking parameters can, in turn, be understood by the mixings of active neutrinos with a sterile neutrino, which by definition does not possess a flavor number, and thus neither respects active flavor symmetries. The model we have elaborated on in the text incorporates the positive features of (perturbative)  $\mu - \tau$  models, allowing at the same time for a natural explanation of LSND/MiniBooNE results. As we have discussed, the model can fix all required parameters using oscillation

neutrino observables. The allowed one sigma parameter space turns out to be narrow, but we consider it a nice feature of the model, since it allows us to explore its prediction without further approximations. In particular, the model is consistent with observed bounds on the sterile fraction in solar neutrino flux and predicts distinctive modifications on the allowed region for the neutrinoless double beta decay parameter,  $m_{ee}$ . From here, EXO limits already impose an upper bound for an absolute neutrino mass at about  $0.5 \text{ eV}$ . Moreover, even though the  $m_{ee}$  region is wider compared to that of three neutrino scenarios, it predicts a lower value for  $m_{ee}$ , which would be reachable in forthcoming experiments. As a matter of fact, our model could be excluded if no positive signal is found above  $|m_{ee}| \sim 0.01 \text{ eV}$ .

Along the analysis, we have not included a Dirac  $CP$ -violating phase. Majorana  $CP$  phases, on the other hand, have been also fixed to  $0$  or  $\pi$  values, which amount only to fixing the relative sign of the mass eigenstates. Nevertheless, it is interesting that our analysis shows that the only consistent combination of relative signs that give appropriated perturbative solutions comes when  $m_1 < 0$ , whereas other masses are positive. This is an intriguing result that could be clarified by an extended exploration of allowed  $CP$  phases in the model. Such an analysis is out of the scope of the present discussion, but it is part of the further work we are already undertaking.

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